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THE MATHEMATICS TEACHER

Volume XXXI Number 2

Edited by William David Reeve

A Course in Applied Mathematics for Teachers of Secondary Mathematics*

By CLEON C. RICHTMEYER
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Mount Pleasant, Michigan

IT HAS long been the feeling of the writer that the present curricula for teachers of secondary mathematics tend more to theory than to applications, and that a need exists for a course in which the many applications of mathematics could be considered. This feeling increased in intensity due to the following trends in teaching of secondary mathematics: (1) A shift in emphasis from disciplinary values to appreciation and application; and (2) The reorganization of secondary mathematics to include a general course for all students. These trends both imply a teacher with a broad background of knowledge of the application of mathematics.

In the fall of 1935, a study was undertaken to discover whether this need was generally felt, and also to determine what applications of mathematics would be most important for secondary teachers to know about. Participating in the study were: (a) 209 teachers of secondary mathematics from thirty-six states, (b) 92 heads of mathematics departments in teachers colleges of the North Central Association, and (c) 17 curriculum specialists regarded as outstanding in the fields of philosophy of education, psychology of learning, general curriculum construction, college curriculum construction, research in secondary education, and research in methods of teaching secondary methematics.

Of group (a), 82 per cent were conscious of a need for more knowledge about applications in mathematics, while 92 per cent agreed that a general course in applications would be valuable for a prospective teacher of secondary mathematics. Eighty-three per cent of group (b) felt that such a course would be valuable for a prospective teacher.

All groups were asked to select items which they considered to be of most importance. The following eight items were agreed upon by all groups as being of primary importance to secondary teachers:

1. Uses of geometry in shop work.
2. Use of the slide rule for approximate computation.
3. Use of surveying instruments in indirect measurement.
4. Use of tables of compound interest and annuities in connection with installment buying, sinking funds, depreciation, and other problems of finance.
5. Use of mathematics in statistical methods.

* Based upon *A Course in Applied Mathematics for Teachers of Secondary Mathematics*. Unpublished Doctor's Field Study, Colorado State College of Education, Greeley, Colo., 1936.

6. Application of mathematics to the social sciences.
7. Application of solid geometry to mensuration.
8. Application of mathematics to agriculture.

The following nine items were selected by one or two of the groups as being of primary importance:

1. Construction and use of graphs including semi-logarithmic graphs.
2. Application of ratios, proportions and percentages to chemistry and physics.
3. Applications of geometry to art and design.
4. Applications of calculus and simple differential equations to physics.
5. Applications of calculus and differential equations to chemistry.
6. Applications of calculus to engineering.
7. Applications of mathematics to biology.
8. Applications of mathematics to pharmacy.
9. Applications of mathematics to army calculations.

The following list of miscellaneous applications was suggested by one or more individuals:

1. Use of sextant in navigation.
2. Mathematics needed in aerodynamics.
3. Applications of complex numbers to the study of vibration and alternating currents.
4. Mathematics in carpentry and building trades.
5. Mathematics of the automobile and internal combustion engines.
6. Mathematics and religion.
7. Use of plane table and alidade in map drawing.
8. Applications to architecture.
9. Applications to music.
10. Use of mathematics in home economics.
11. Applications of mathematics to study of forestry.
12. Applications of mathematics in radio.
13. Applications of solid geometry and analytics to astronomy.
14. Applications of mathematics to study of forces in physics.
15. Measuring production and consumption.
16. Problems of transportation and communication.
17. Computing the Federal income tax.
18. Problems involving use of cubic and quartic equations.
19. Use of mathematics in insurance.
20. The mathematics and use of the polar planimeter.
21. Use of simple instruments such as tape or strong cord in direct measurements in laying out ball diamond, tennis courts, etc.
22. Algebraic equations, their graphs and use in design.
23. Linkages.
24. Applications of theory of probability to games of chance, lotteries, insurance, weather reports, etc.
25. Study of the adaptations of nature to meet needs of maximum and minimum forces: in shapes of butterflies, insects, fishes, ships, airplanes, raindrops, trees, architectural structures, shortest paths, paths for shortest time.

Based upon the criteria for general teacher education objectives, as set forth in the National Survey of the Education of Teachers,[1] the following objectives for a course in applied mathematics for teachers were set up:

1. To give the teacher a broad background of knowledge of how and where mathematics functions in the world's work.
2. To help the prospective teacher to see an integrated picture of the whole structure of mathematics including its uses in the modern world.
3. To give the teacher a better background for the new courses in general mathematics now being introduced in the secondary schools.
4. To enable the teacher to make the traditional courses in mathematics more interesting and inspiring.
5. To develop the habit of reading books and periodicals to discover applications of mathematics.
6. To develop the ability of the teacher to discover applications in every day experiences of himself and others.
7. To provide opportunity for individual investigation.

The course was organized into units based upon the topics agreed upon by all

[1] Rugg, Earle U., et al., *National Survey of the Education of Teachers*, U. S. Education Bureau Bulletin, 1933, No. 10, Vol. III.

groups as being of primary importance. Other topics mentioned may be investigated by students having particular interest in these fields. Listed below under each unit, are found typical problems and suggested problems for individual investigation.

Unit One[2]

Use of the Slide Rule in Approximate Computation

A. General References.

1. Clark, J. J., *The Slide Rule*. Frederick J. Drake and Company, Chicago, 1929.
2. Cobb, H. E., *Applied Mathematics*, Chapter XI, Ginn and Company, Boston, 1911.
3. Keuffel and Esser, *Manual of the Slide Rule*. Pamphlet published by Keuffel and Esser Company, New York.
4. Marsh, H. W., *Industrial Mathematics*, Chapter XV, John Wiley and Sons, 1913.
5. Marsh, H. W., *Technical Algebra*, Chapter XVI, John Wiley and Sons, New York, 1913.
6. Perry, John, *Elementary Practical Mathematics*, Chapter III, Macmillan, New York, 1913.
7. Rasor, S. E., *Mathematics for Students of Agriculture*, pp. 133–138, Macmillan, New York, 1921.
8. Richtmeyer, C. C. and Foust, J. W., *Business Mathematics*, pp. 78–81, McGraw-Hill, New York, 1936.
9. Shuster, C. N. and Bedford, F. L., *Field Work in Mathematics*, Chapter V, American Book Company, 1935.
10. Slade, Samuel and Margolis, Louis, *Mathematics for Technical and Vocational Schools*, Chapter VII, John Wiley and Sons, New York, 1922.
11. Strohm, R. T., *The Slide Rule and How to Use It*. International Textbook Company, Scranton, Pa., 1924.

B. Illustrative Problems.

1. 21.4×3.27
2. 750×21.6
3. $89.7 \div 14.2$
4. $5.63 \div 64.1$
5. $\dfrac{41 \times 69 \times 21.7}{59.1 \times 6.27}$
6. *8 is what per cent of 24*
7. *Find two per cent of $197*
8. *173 is 6% of what number?*
9. *Find square root and cube root of .0234*
10. *Find x in the proportion* $\dfrac{x}{72} = \dfrac{482}{173}$
11. *Find the area and circumference of a circle whose radius is 7.5 inches.*
12. *If the wages of 3 men for one day are $16.50 what are the wages of 14 men at the same rate?*
13. *An article costing 9 cents is sold at two for 25¢. Find the per cent of profit on the cost; on the selling price.*
14. *Find the radius of a sphere whose volume is 100 cu. ft.*
15. *How many tons of sea water are necessary to produce twenty-five pounds of salt, if sea water is 2.71% salt?*
16. *Find sine, cosine, and tangent of 50° 40′.*
17. *Find sine, cosine, and tangent of 32° 15′.*
18. *If, at a point 150 feet measured horizontally from the base of a building, the angle of elevation of the top is 40° 20′; find the height of the building.*
19. *A telephone pole 60 feet high is to be braced by a guy wire making an angle of 35° with the pole. How long will the guy wire need to be?*
20. *From the top of a light house 100 feet high, standing on a cliff, the angle of depression of a ship was 4° 30′, and at the bottom of the lighthouse the angle of depression was 3° 20′. Find the horizontal distance of the ship from the lighthouse.*

C. Problems for Individual Investigation.

1. Discuss the different types of slide rules and their uses.
2. Construct a slide rule.
3. Make a list of one hundred percentage problems and compute both by arithmetic and by the slide rule. Compare the length of time necessary for computation by the two methods, and compare the accuracy.

[2] In this and the following units it is not intended to present a complete bibliography. Limited space obviously prevents such an exhaustive list. The plan is to present a representative list of references which may be readily supplemented in most cases by other references of equal quality.

4. Write a history of the development of the slide rule.

Unit Two

Use of Geometry in Shop Work and Mensuration

A. General References

1. Barker, E. H., *Applied Mathematics*, Chapter IX, Allyn and Bacon, New York, 1920.
2. Christman, J. M., *Shop Mathematics*, Chapters XVII–XXIV, Macmillan, New York, 1926.
3. Cobb, H. E., *Elements of Applied Mathematics*, Chapters IX and XV, Ginn and Company, Boston, 1911.
4. Farnsworth, P. V., *Industrial Mathematics*, Parts I and II, D. Van Nostrand Company, New York, 1930.
5. Hale, J. W. L., *Practical Applied Mathematics*, Chapters VII–X, XVI–XVIII, McGraw-Hill, New York, 1915.
6. Johnson, James F., *Practical Shop Mechanics and Mathematics*, Chapters II and III, John Wiley and Sons, New York, 1926.
7. Kern, W. F. and Bland, J. R., *Solid Mensuration*. John Wiley and Sons, New York, 1934.
8. Leonard, C. J., "Mathematics in Industry," *School Science and Mathematics*, Vol. 32, pp. 745–747 (1932).
9. Marsh, H. W., *Industrial Mathematics*, Chapters VI, VII, and XI, John Wiley and Sons, New York, 1913.
10. Meyer, J. A. and Sampson, C. H., *Practical Trade Mathematics*, Chapters VII and IX, New York, 1920.
11. Norris, E. B. and Craigo, R. T., *Advanced Shop Mathematics*, Chapters X–XVII, McGraw-Hill, New York, 1913.
12. Perry, John, *Elementary Practical Mathematics*, Chapters VI, X, XI, Macmillan, New York, 1913.
13. Slade, Samuel and Margolis, Louis, *Mathematics for Technical and Vocational Schools*, Chapters V, XII, XIII, John Wiley and Sons, New York, 1922.
14. Wolfe, J. H. and Phelps, E. K., *Practical Shop Mathematics*, Chapter X, McGraw-Hill, New York, 1935.

B. Illustrative Problems.

1. *How would you cut a regular hexagonal top for a plant stand from a piece of wood 18″ by 18″? What would be the length of each side?*
2. *The same as problem one, except that the top is to be a regular octagon instead of a hexagon.*
3. *A foot stool with a circular top 15″ in diameter is to have three legs placed so that their centers are one inch inside the circumference of the top. How far apart should the centers of the legs be?*
4. *What is the largest perfect square that can be milled on a shaft 1.5″ in diameter?*
5. *Calculate the size of a circle required in which to turn a square nut whose side is ¾ inch.*
6. *A horizontal cylindrical boiler is 45 inches in diameter and 12 feet long. If it is filled with water to a depth of 30 inches, what is the volume of the steam space remaining above the water line?*
7. *How many ounces of nickel would be used in plating one gross balls 1.5 inch in diameter to a depth of 1/64 inch? (Nickel weighs 5.14 oz. per cubic inch).*
8. *A steel disc 12 inches in diameter and ½ inch thick has four one-inch holes drilled in it to lighten the weight. If the steel weighs .28 pound per cubic inch find how much the weight of the disc has been reduced.*
9. *How many square feet of tin 3 feet wide and ⅛″ thick are there in a roll weighing 100 pounds? Tin weighs .26 pounds per cubic inch.*
10. *How many feet of copper wire ⅛″ in diameter can be drawn from a bar of copper two inches square and six feet long?*
11. *What must be the diameter of a pipe to have the same carrying capacity as two pipes of diameter two and three inches respectively?*
12. *A pail 14 inches high has top and bottom diameters of 12 and 8 inches respectively. Find (a) the capacity in gallons, (b) the number of square inches of sheet metal required to make the pail.*
13. *A swimming pool has a rectangular top 30 by 100 ft. The ends and sides are vertical and the bottom slopes gradually from a depth of 4 feet at one end to a depth of 10 feet at the other. Find the volume of water necessary to fill the pool.*
14. *A circular table top 50 inches in diameter and ¾ inch thick is to be made from solid oak. How heavy will it be if oak weighs 52 pounds per cubic foot?*

15. *A lamp shade frame is built in the form of a frustrum of a cone, the diameters of the bases being 7 and 15 inches respectively, and a slant height of 12 inches. How much material is necessary to cover it if $\frac{1}{2}$ inch is allowed for the seam?*
16. *A rivet is to be forged from a steel rod $\frac{1}{2}$ inch in diameter, so that the head is a hemisphere of diameter one inch. What must be the original length of the rod in order that the completed rivet shall have a shank four inches long?*
17. *A circular plate 10 inches in diameter is to have five bolt holes drilled so as to be equally spaced on a circle 1.5 inches inside the edge of the plate. How would you locate the centers of the bolt holes? How far apart would consecutive holes be?*
18. *A spindle, the diameter of the large end of which is one inch and whose length is sixteen inches, is to be turned with a taper of .375 inch per foot. What will be the diameter of the small end and how much should the tail stock be offset?*
19. *From a rectangular piece of tin 16 inches by 32 inches a box is to be made by cutting out equal squares from the corners and turning up the sides.*
    (a) *Find the volume of the box if two-inch squares are cut out.*
    (b) *Find the volume if five-inch squares are cut out.*
    (c) *Find the side of square cut out which will give a box of the largest volume.*

C. Problems for Individual Investigation.

1. Derive a formula for the length of an open belt around two pulleys of different sizes.
2. Derive a formula for the length of a crossed belt around two pulleys of different sizes.
3. Select a textbook in some phase of industrial arts such as woodwork, sheet metal work, pattern making, etc., and list the geometrical facts and concepts necessary to read the text intelligently.

Unit Three

Use of Surveying Instruments in Indirect Measurement

A. General References.

1. Breed, C. B. and Hosmer, G. L., *The Principles and Practice of Surveying.* John Wiley and Sons, New York, 1908.
2. Davis, R. E., *Elementary Plane Surveying.* McGraw-Hill, New York, 1936.
3. Ormsby, M. T. M., *Elementary Principles of Surveying.* Scott, Greenwood and Son, London, 1925.
4. Pence, W. D. and Ketchum, M. S., *Surveying Manual.* McGraw-Hill, New York, 1932.
5. Shuster, C. N. and Bedford, F. L., *Field Work in Mathematics,* Chapters X, XI, XIV, American Book Company, 1935.
6. Tracy, J. C., *Plane Surveying and Exercises in Surveying.* John Wiley and Sons, New York, 1907.

B. Illustrative Problems.

1. *Using the transit find the height of a church spire or tall chimney.*
2. *Using the transit find the width of a river.*
3. *Using the transit find the area of a field.*
4. *Using the transit find the horizontal distance between two points on opposite sides of a building or hill.*
5. *Using the level find the fall in water level between two points on a river.*
6. *Using the transit and stadia rod make a contour map of a field, taking readings every fifty feet.*

C. Problems for Individual Investigation.

1. Write a history of the U. S. Public Land Survey.
2. Write out in detail the methods of surveying used in dividing land into townships, sections, and smaller subdivisions.
3. Discuss different types of surveying instruments and their uses.

Unit Four

Applications of Mathematics to Agriculture

A. General References.

1. Barker, E. H., *Applied Mathematics,* Chapter XIII, Allyn and Bacon, Boston, 1920.

2. Plant, L. C., *Agricultural Mathematics.* McGraw-Hill, New York, 1930.
3. Rasor, S. E., *Mathematics for Students of Agriculture.* Macmillan, New York, 1921.
4. Roe, H. B., "Mathematics in Agriculture," *Sixth Yearbook*, National Council of Teachers of Mathematics, Teachers College, New York, 1931.
5. Roe, H. B., Smith, D. E. and Reeve, W. D., *Mathematics for Agriculture and Elementary Science.* Ginn and Company, Boston, 1928.

B. Illustrative Problems.

1. *How many pounds of skim milk must be added to 200 pounds of milk containing 5% butter fat to standardize it to 3% milk?*
2. *A ton of fertilizer for corn is to be made containing 3% nitrogen, 8% phosphoric acid, and 4% potash from Chilean nitrate of soda containing 15% nitrogen, acid phosphate containing 16% phosphoric acid, and muriate of potash containing 50% potash. How much of each ingredient should be used and how much filler will be needed?*
3. *An open ditch is 20 inches wide at the top, 16 inches at the bottom, and 18 inches deep. If the rate of flow is 3 feet per minute, what is the maximum number of gallons of water the ditch can carry in one minute?*
4. *The butter fat record for a cow over a period of one week is given below.*

Milking	1	2	3	4	5	6	7
Per cent of butter fat	5.5	4.7	4.3	2.9	3.3	4.1	4.0

Milking	8	9	10	11	12	13	14
Per cent of butter fat	3.5	3.5	3.4	2.9	1.9	4.9	4.3

Represent the data graphically and compare with a 3% "legal minimum" line.

5. *The number of tons of ensilage in a 14 foot silo is given by the formula* $W = .0405\, d^2 + 1.82\, d$, *where d is the depth in feet. How many tons of ensilage remain in a 14 foot silo which was originally filled to a depth of 40 feet, if 25 feet has been used from the top?*
6. *If one acre of corn will produce fifteen tons of ensilage, how many acres of corn should be planted to fill two 14 foot silos, 30 and 40 feet high respectively?*
7. *The number of kilo-pound units of energy needed in the food of sheep has been found empirically to be given by formula,* $K = W^{.7}/10^{.98}$ *where W is the weight of the sheep in pounds. Find the number of kilo-pound units needed for a 140 pound sheep.*
8. *A farmer is offered 50¢ a bushel for his corn in the fall. If he holds it six months what price must he then receive if the corn shrinks 12% and money is worth 5%?*
9. *A ton of a certain fertilizer consists of 50% acid phosphate, 25% dried blood, and 10% muriate of potash. How many pounds of acid phosphate must be added to give a fertilizer having a* $62\frac{1}{2}$% *acid phosphate content?*
10. *What quantities of sodium nitrate (15% nitrogen), bone meal (3% nitrogen, 24% phosphoric acid), and wood ashes (1% phosphoric acid, 5% potash) are needed for an acre of buckwheat requiring 15 pounds of nitrogen, 30 pounds phosphoric acid, and 35 pounds of potash per acre?*
11. *Under certain atmospheric conditions bacteria in milk double in number every twenty minutes. If at a certain time there are 5000 bacteria in a cubic centimeter of milk, how many will be present six hours later?*
12. *A farmer has 100 rods of fence with which he wishes to make a temporary rectangular pasture along side an existing line fence. What will be the dimensions and area if the largest possible area is enclosed?*
13. *Using a tape and level set out foundation stakes for a barn.*
14. *Using a transit make a profile for a tile drain for a field.*

C. Problems for Individual Investigation.

1. Interview a number of progressive

farmers to learn what mathematics they make use of.

2. Investigate the application of the theory of probability to the study of heredity.

UNIT FIVE

Use of Tables of Compound Interest and Annuities in Problems of Finance

A. General References.

1. Glover, J. W., *Tables of Applied Mathematics in Finance, Insurance, Statistics.* George Wahr, Ann Arbor, 1923.
2. Hart, W. L., *Brief College Algebra,* Chapter XVI, D. C. Heath, Boston, 1932.
3. Hart, W. L., "The Mathematics of Investment," *Sixth Yearbook,* National Council of Teachers of Mathematics, Teachers College, Columbia, New York, 1931.
4. Hart, W. L., *Mathematics of Investment.* D. C. Heath, Boston, 1929.
5. Richtmeyer, C. C. and Foust, J. W., *Business Mathematics,* Chapters III, VI, VII, McGraw-Hill, New York, 1936.
6. Skinner, E. B., *Mathematical Theory of Investment.* Ginn and Company, Boston, 1924.
7. Smail, L. L., *Mathematics of Finance.* McGraw-Hill, New York, 1934.
8. Toner, J. V., *Mathematics of Finance.* Ronald Press, New York, 1926.

B. Illustrative Problems.

1. *The cash price of a washing machine is $65, but it can be purchased for $10 down and $10 a month for 6 months. What is the simple rate of interest charged?*
2. *Mr. Smith buys a house valued at $10,000 paying $3,000 down. He agrees to pay $2,000 in 2 years, another $2,000 at the end of four years, and the remainder at the end of 7 years. What is the final payment if money is worth 5% compounded annually?*
3. *Small loan companies in Michigan may legally charge $3\frac{1}{2}$% per month. This is equivalent to what annual rate?*
4. *A father wishing to provide a college education for his son, begins in the year of his son's birth to set aside a certain sum at the end of each year. If $2,500 is desired on the son's eighteenth birthday how much should be set aside at the end of each year, provided the fund accumulates at 4% compounded annually?*
5. *A loan of $6,000 made to build a house, is to be paid back in equal monthly installments over a period of 15 years. If interest at 6% compounded monthly is charged, what is the necessary monthly payment?*
6. *A motor bus costs $10,000 and will last five years, having a scrap value of $1,000. Under the sinking fund plan of depreciation, using interest rate of 5% find (a) the annual replacement charge; (b) the amount in the sinking fund at the end of three years. Construct a complete depreciation schedule.*
7. *A certain college has running expenses of $300,000 a year. What endowment invested at $4\frac{1}{4}$% would be necessary to provide these expenses indefinitely?*
8. *A mile of pavement costing $35,000 must be completely rebuilt every 25 years. If in addition $500 per year must be spent for maintenance, find the capitalized cost if money is worth 3% compounded annually.*
9. *A $1,000 4% bond with annual dividends is to be redeemed at par in 15 years. Find the purchase price to yield the investor (a) 3%; (b) 4%; (c) 5%.*
10. *The premium on a 20 year endowment policy for $10,000 is $452.30 payable annually in advance. If the payments had been invested at the beginning of each year in a savings fund paying $2\frac{1}{2}$% compounded annually, how much would the fund contain at the end of 20 years?*
11. *How long will it take to pay off a loan of $5,000 by installments of $50 per month if 6% interest compounded monthly is charged?*
12. *How much would the time in problem 11 be shortened if the interest rate were 4%?*
13. *How long could a life insurance company guarantee an income of $100 per month to the beneficiary of a $10,000 policy if the company earns 3% compounded monthly?*

14. A government bond costing $75 has a maturing value of $100 in ten years. What nominal interest rate is earned for the investor if interest is assumed to be compounded annually? semi-annually? quarterly?

C. Problems for Individual Investigation.

1. Making use of expansion by series, derive an approximate formula of three terms for the time needed for money to double itself at a rate of r per period.
2. Investigate the mathematics involved in conducting a building and loan company.
3. Investigate at your local bank, the FHA plan of loans for building and modernizing. Write a complete account of the mathematics involved.
4. Investigate the mathematics involved in the government rural electrification plan.

Unit Six

Uses of Mathematics in Statistical Methods

A. General References.

1. American Statistical Association, "Collegiate Training in Mathematical Statistics," *Mathematical Monthly*, Vol. 41, pp. 598–599. (Dec. 1934).
2. Brown, Ralph, *Mathematical Difficulties of Students of Educational Statistics.* Teachers College, Columbia, New York, 1933.
3. Camp, B. B., *The Mathematical Part of Elementary Statistics.* D. C. Heath, Boston, 1931.
4. Chaddock, R. E. and Croxton, F. E., *Exercises in Statistical Methods.* Houghton-Mifflin, Boston, 1924.
5 Cooke, D. H., *Minimum Essentials of Statistics.* Macmillan, New York, 1936.
6. Hart, W. L., *Brief College Algebra*, Chapter XXII, D. C. Heath, Boston, 1932.
7. Kelley, T. L., *Statistical Method.* Macmillan, New York, 1923.
8. Peters, C. C., *Statistical Procedures and Their Mathematical Bases.* Pennsylvania State College, State College, Pa., 1935.
9. Richtmeyer, C. C. and Foust, J. W., *Business Mathematics*, Chapter X, McGraw-Hill, New York, 1936.
10. Rietz, H. L., *Handbook of Mathematical Statistics.* Houghton-Mifflin, Boston, 1924. (Includes an extended bibliography.)
11. Rietz, H. L., *Mathematical Statistics.* Open Court Publishing Company, Chicago, 1927.
12. Rugg, H. O., *Statistical Methods Applied to Education.* Houghton-Mifflin Company, Boston, 1917.
13. Walker, Helen M., "Mathematics and Statistics," *Mathematics in Modern Life.* Sixth Yearbook, National Council of Teachers of Mathematics, Teachers College, Columbia, New York, 1931.
14. Walker, Helen M., *Mathematics Essential for Elementary Statistics.* Henry Holt, New York, 1934.
15. West, C. J., *Introduction to Mathematical Statistics.* R. G. Adams and Company, Columbus, 1918.
16. Whitney, F. L., *Statistics for Beginners in Education.* D. Appleton and Company, New York, 1929.
17. Yule, G. U., *An Introduction to the Theory of Statistics.* Charles Griffin and Company, London, 1919.

B. Illustrative Problems (Limited to that part of statistics commonly designated as elementary statistics).

Table I

Intelligence Quotients of Twenty-Eight Students and Their Scores on an Algebra Test

Student Number	IQ	Test Score	Student Number	IQ	Test Score
1	121	72	15	122	84
2	116	62	16	129	91
3	111	70	17	104	71
4	120	73	18	123	68
5	139	73	19	113	54
6	111	74	20	111	82
7	129	77	21	120	78
8	110	84	22	111	67
9	123	84	23	115	61
10	128	73	24	103	56
11	118	73	25	106	67
12	116	72	26	127	78
13	126	78	27	113	69
14	122	78	28	127	62

In the following problems use the data of Table I.

1. *Arrange the data of the second and third columns of Table I into frequency distributions with a step of 3. Show results graphically by means of histograms and frequency polygons.*
2. *Compute the arithmetic mean, median, mode for the distributions.*
3. *Compute the quartiles, percentiles, percentile ranks, and construct ogive curves.*
4. *Determine the quartile deviation, average deviation, and standard deviation of the distributions.*
5. *Determine the skewness of the distributions.*

6. *Rank the students according to their scores in the two columns in the table and compute the coefficient of correlation by the Spearman ranks method.*
7. *Construct a scattergram and compute the coefficient of correlation by the Pearson product-moment method.*
8. *Using the method of least squares, find the equation of the line of best fit. Show graphically the relation between the line and the data.*

C. Problems for Individual Investigation.

1. Analyze in detail one of the above problems including the development of any necessary formulas, and list the mathematical concepts and operations involved.
2. Toss 10 pennies, 100 times, recording the number of heads and tails appearing on each trial. Construct a frequency polygon, showing the distribution of the combinations of the heads and tails.
3. Work through a development of the equation of the normal probability curve, listing the mathematical concepts and processes involved.
4. Investigate the mathematics of partial and multiple correlation.

Unit Seven

Applications of Mathematics to the Social Sciences

A. General References.

1. American Statistical Association, *Statistics in Social Studies.* University of Pennsylvania Press, 1930.
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3. Cobb, J. C., "Quantitative Restating of Sociological and Economic Problems," *American Journal of Sociology,* Vol. 32, pp. 921–930 (May 1927).
4. Ellwood, C. A., "The Uses and Limitations of the Statistical Method in the Social Sciences," *Scientific Monthly,* Vol. 37, pp. 353–357 (Oct. 1933).
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14. Lefever, D. Welty, "Measuring Geographic Concentration by Means of the Standard Deviational Ellipse," *American Journal of Sociology,* Vol. 32, pp. 88–94 (May 1927).
15. Mills, F. C., *Statistical Methods Applied to Economics and Business.* Henry Holt, New York, 1924.
16. Morris, C. C., "Mathematical Methods in Economic Research," *American Mathematical Monthly,* Vol. 31, pp. 57–64 (Feb. 1924).
17. Richtmeyer, C. C. and Foust, J. W., *Business Mathematics,* Chapter X, McGraw-Hill, New York, 1936.
18. Secrist, Horace, *Statistics in Business.* McGraw-Hill, New York, 1920.
19. Tolley, H. R., "Collegiate Mathematics Needed in the Social Sciences," *American Mathematical Monthly,* Vol. 39, p. 503, and pp. 569–577 (Dec. 1932).

B. Illustrative Problems.

1. *Find from the Statistical Abstract of the United States, the average daily number of telephone messages for the last four years given. Plot these data on a time scale and find the equation of the trend line. Predict the number of messages for the following year.*
2. *Obtain from the current Statistical Abstract of the United States, the average weekly wages of cutters in the boot and shoe industry, from 1913 to date. Find also the average prices of potatoes per bu. for the same years. Using 1913 as a base compute index numbers of the*

wages and prices for each year and show graphically, by means of line graphs, the relation between the two sets of index numbers. Determine the ratio between the amounts of potatoes that a week's wage would buy in 1913 and 1920; 1913 and 1930; 1913 and the last year for which data are available.

3. *Using a table of mortality compute the probability that an individual 20 years of age will live to be 60. What sum should he pay now for a pure endowment of $10,000 maturing at age 60?*
4. *Find the correlation between the number of deaths per 100,000 population caused by automobile accidents in each state and the average population per square mile.*
5. *In establishing a new exchange a telephone company charges $50 per subscriber for 100 subscribers or less. In order to obtain a larger number of subscribers the company agrees to reduce the price per subscriber by 25¢ for each additional subscriber above 100. In order to produce the maximum income for the company, how many subscribers should be obtained?*

C. Problems for Individual Investigation.

1. Investigate the mathematics of the Cournot theory of monopoly.
2. Investigate the mathematics of the equation of exchange.
3. Discuss the various types of index numbers and their uses.
4. Choose one of the following topics and develop a unit for a ninth grade class in general mathematics.
   (a) Installment buying.
   (b) Comparative costs of owning and renting a home.
   (c) Budgeting income.
   (d) Investments.
   (e) Insurance.
   (f) Taxation.
   (g) Social security.
   (h) Credit and banking.
   (i) Vital statistics.
   (j) Cost of owning a car.
   (k) Statistical graphs.

SUPPLEMENTARY APPLICATIONS

I. Construction and Use of Graphs Including Semi-logarithmic Graphs.

A. References.

1. Brinton, W. C., *Graphic Methods for Presenting Facts.* Engineering Magazine Company, New York, 1931.
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3. Karsten, Karl G., *Charts and Graphs.* Prentice-Hall, New York, 1923.
4. Richtmeyer, C. C. and Foust, J. W., *Business Mathematics.* Chapter VIII, McGraw-Hill, New York, 1936.
5. Riggleman, John R., *Graphic Methods for Presenting Business Statistics.* McGraw-Hill, New York, 1932.

II. Applications of Ratios, Proportions, and Percentages to Chemistry and Physics.

A. References.

1. Boles, L. L. and Webb, H. A., "Mathematics of General Inorganic Chemistry," *Science Education,* Vol. 14, pp. 539–546 (March 1930).
2. Brown, H. Emmet, "Mathematics in Physics," *Sixth Yearbook,* National Council of Teachers of Mathematics, Teachers College, Columbia, New York, 1931.
3. Daniels, Farrington, "The Mathematical Training of Chemists," *American Mathematical Monthly,* Vol. 40, pp. 1–4 (Jan. 1933).

III. Applications of Geometry to Art and Design.

A. References.

1. Birkhoff, George D., "Polygonal Forms," *Sixth Yearbook,* National Council of Teachers of Mathematics, Teachers College, Columbia, New York, 1931.
2. Ewer, A. Bruce, *Geometric Design.* Smith-Brooks, Denver, 1936.
3. Schorling, Raleigh and Clark, J. R., *Mathematics in Life,* Unit B, World Book Company, New York, 1935.
4. Weir, B. L., "Relating Art and Mathematics," *Mathematics Teacher,* Vol. 23, p. 60 (Jan. 1930).

IV. Applications of Calculus and Simple Differential Equations to Physics.

A. References.

1. Hedrick, E. R. and Ke'log, O. D., *Applications of the Calculus to Mechanics.* Ginn and Company, Boston, 1909.
2. Norris, P. W. and Legge, W. S., *Mechanics Via the Calculus.* Longmans, Green and Company, London, 1923.

3. Stern, W. A., "The Role of Mathematics in Modern Physical Theory," *The Monist*, Vol. 39, pp. 263–272 (1929).

V. Applications of Calculus and Simple Differential Equations to Chemistry.

A. References.

1. Daniels, Farrington, *Mathematical Preparation for Physical Chemistry*. McGraw-Hill, New York, 1929.
2. Daniels, Farrington, "Mathematics for Students of Chemistry," *American Mathematical Monthly*, Vol. 35, pp. 3–9 (Jan. 1928).
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4. Fernelius, W. C., "Applications of Mathematics to Chemistry," *School Science and Mathematics*, Vol. 29, pp. 71–78 (Jan. 1929).
5. Hitchcock, F. L. and Robinson, C. S., *Differential Equations in Applied Chemistry*. John Wiley and Sons, New York, 1923.

VI. Applications of Calculus to Engineering.

A. References.

1. Cole, H., "Practical Application of Mathematics to Engineering Problems," *The Mathematics Teacher*, Vol. 25, pp. 290–297 (May 1932).
2. Dudley, A. M., "The Type of Mathematical Training Needed by Electrical Engineers," *American Mathematical Monthly*, Vol. 42, pp. 301–306 (May 1935).
3. Ellis, C. A., "Mathematics and Engineering Education," *School Science and Mathematics*, Vol. 35, pp. 123–132 (Feb. 1935).
4. Howland, W. E., "Mathematics in Civil Engineering," *School Science and Mathematics*, Vol. 35, pp. 351–360 (April 1935).
5. Karepetoff, V., *Engineering Applications of Higher Mathematics*. John Wiley and Sons, New York, 1916.
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7. Steinmetz, C. P., *Engineering Mathematics*. McGraw-Hill, New York, 1917.

VII. Applications of Mathematics to Biology.

A. References.

1. Buchanan, H. E., "A Mathematical Theory of the Transmission of Successive Impulses Through a Muscle," *American Mathematical Monthly*, Vol. 37, pp. 219–224 (May 1930).
2. Harris, J. A., "Mathematics in Biology," *Sixth Yearbook*, National Council of Teachers of Mathematics, Teachers College, Columbia, New York, 1931.
3. Harris, J. A., "The Fundamental Mathematical Requirements of Biology," *American Mathematical Monthly*, Vol. 36, pp. 179–198 (April 1929).
4. Richards, O. W., "The Mathematics of Biology," *American Mathematical Monthly*, Vol. 32, pp. 30–36 (Jan. 1925).

VIII. Applications of Mathematics to Pharmacy.

A. References.

1. Spease, Edward, "Mathematics in Pharmacy and Allied Professions," *Sixth Yearbook*, National Council of Teachers of Mathematics, Teachers College, Columbia, New York, 1931.
2. Spease, Edward, *Pharmaceutical Mathematics*. McGraw-Hill, New York, 1930.

IX. Applications of Mathematics to Army Calculations.

A. References.

1. Grenhill, Sir G., "Mathematics in Artillery Science," *Mathematical Gazette*, Vol. 8, p. 25 (March 1915).
2. Marsh, H. W., *Technical Trigonometry*, p. 150, John Wiley and Sons, New York, 1914.
3. Sherrill, C. O., *Military Topography*. George Banta Publishing Company, Menasha, Wis., 1912.

MISCELLANEOUS APPLICATIONS

1. Use of sextant in navigation.
2. Mathematics needed in aerodynamics.
3. Applications of complex numbers to the study of vibrations and alternating currents.
4. Mathematics in carpentry and building trades.
5. Mathematics of the automobile and internal combustion engines.
6. Mathematics and religion.
7. Use of plane table and alidade in map drawing.
8. Applications to architecture.
9. Applications to music.
10. Use of mathematics in home economics.
11. Applications of mathematics to study of forestry.
12. Applications of mathematics in radio.
13. Applications of solid geometry and analytics to astronomy.
14. Applications of mathematics to a study of forces in physics.

15. Measuring production and consumption.
16. Problems of transportation and communications.
17. Computing the Federal income tax.
18. Problems involving use of cubic and quartic equations.
19. Use of mathematics in insurance.
20. The mathematics and use of the polar planimeter.
21. Use of simple instruments such as tape or strong cord in direct measurements in laying out ball diamonds, tennis courts, etc.
22. Algebraic equations, their graphs and use in design.
23. Linkages.
24. Applications of theory of probability to games of chance, lotteries, insurance, weather reports, etc.
25. Study of the adaptation of nature to meet needs of maximum and minimum forces; in shapes of butterflies, insects, fishes, ships, airplanes, raindrops, trees, architectural structures shortest paths, paths for shortest time.

Miscellaneous References

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2. Blake, Sue Avis, "Some of Nature's Curves," *School Science and Mathematics*, Vol. 36, pp. 245–249 (March 1936).
3. Bliss, G. A., "Mathematical Interpretations of Geometrical and Physical Phenomena," *American Mathematical Monthly*, Vol. 40, pp. 472–489 (Oct. 1933).
4. Bliss, G. A., "Mathematics," *Scientific Monthly*, Vol. 24, pp. 308–319 (April 1927).
5. Bragdon, Claude, *Architecture and Democracy*. Alfred A. Knopf, New York, 1918.
6. Jackson, Dunham, "The Human Significance of Mathematics," *American Mathematical Monthly*, Vol. 35, pp. 406–411 (Oct. 1928).
7. Keyser, C. J., "The Humanistic Bearing of Mathematics," *Sixth Yearbook*, National Council of Teachers of Mathematics, Teachers College, Columbia, New York, 1931.
8. Koch, E. H., *The Mathematics of Applied Electricity*. John Wiley and Sons, New York, 1912.
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10. Moore, C. N., "Mathematics and the Future," *The Mathematics Teacher*, Vol. 22, pp. 203–214.
11. Moore, C. N., "Mathematics and Science," *Science new Series*, Vol. 81, no. 2089, pp. 27–32 (Jan. 11, 1935).
12. Pershing, A. V., "Single Formula for the Concave and Convex Lenses Takes Place of Four Others," *School Science and Mathematics*, Vol. 34, pp. 929–931 (Dec. 1934).
13. Phillips, E. C., "Some Applications of Mathematics to Architecture; Gothic Tracery Curves," *American Mathematical Monthly*, Vol. 33, pp. 361–368 (1926).
14. Schlauch, W. S., "Mathematics as an Interpreter of Life," *Third Yearbook*, National Council of Teachers of Mathematics, Teachers College, Columbia, New York, 1928.
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17. Smith, D. E., "Mathematics in Training for Citizenship," *Third Yearbook*, National Council of Teachers of Mathematics, Teachers College, Columbia, New York, 1928.
18. Straley, H. W., "The Deficiency in Mathematical Training Required of Geologists," *School Science and Mathematics*, Vol. 32, pp. 745–747 (Oct. 1932).
19. Treutlin, T. E., "Teaching One Theorem in Plane Geometry by the Project Method," *School Science and Mathematics*, Vol. 29, pp. 180–189 (Feb. 1929).
20. Wilson, D., "Mathematics and Home Economics," *School*, Vol. 22, pp. 418–423 (Jan. 1934).

It is recommended that the course be conducted on a laboratory basis. If a student has had an elective course adequately covering one of the primary units, he should not be expected to do the same work as other members of the class, but might carry on some of the original investigations suggested or work on one of the supplementary topics in which he might be interested.

As the course is taught, undoubtedly other items will be added and a more comprehensive list of problems compiled. It is hoped, however, that this study may be a start in the right direction and that it will be of value in meeting the need for better education of teachers of secondary mathematics.

An Incidental or an Organized Program of Number Teaching?*

By C. L. Thiele
Director of Exact Sciences Detroit Public Schools

The question of an incidental or an organized program of number teaching is being answered in various ways. In the main American schools are following organized programs of instruction in arithmetic. There is, however, a marked tendency to place the teaching of numbers on an incidental basis in the primary grades. In certain centers this practice is being extended throughout the elementary school years. In that the question seems to be receiving so much consideration at the present time, a discussion of the two programs would seem to be timely.

The extreme incidental program of number teaching is one which has not been predetermined by teacher or pupil but which aims to utilize the natural interests of pupils in numbers as they occur in their classroom activities. Number learning in such a program is a by-product of the activities in which children engage. Briefly, it is a program in which things happen without much design. Perhaps such a program does not exist in actual practice.

An organized program of teaching may be defined as one for which a course of study has been determined and outlined in advance and is followed more or less closely in the classroom. The fact that the program has been planned in advance does not preclude the possibility of utilizing opportunities for motivation nor the use of "true-to-life" situations in the teaching of numbers. The distinct difference between the two programs is in the degree to which they are planned in advance. For example, the teacher on the organized program may make the acquisition of certain number concepts, facts, skills and abilities one direct aim of teaching but not to the exclusion of other desirable aims. On the other hand, the chief concern of the teacher on the incidental program is to satisfy specific inclinations and interests of children by providing opportunities for participation in classroom activities largely initiated by pupils, irrespective of the control gained over any specific knowledge or skills.

The differences between the two programs in action can perhaps be made clearer through an illustration. For example, by any one of several methods of stimulation the interests of children may be directed to the automobile speedometer, its use and its operation. A speedometer may be examined and its workings demonstrated. Attention may, furthermore, be focused upon the red numbers at the extreme right of the speedometer. Thus pupils may discover that there are ten tenths in a whole and thereby learn something of social value as a result of an organized and directed program.

In contrast with this, consideration may be given to a classroom in which learning is acquired incidentally. During the school year pupils have contacts with the automobile and automobile travel, in their construction work, in travelling in school busses, in discussions about speed of travel, and the like. In the course of events it is possible that attention becomes centered upon the operation of the speedometer. If so, the questions of those interested therein are answered, the answers being heard by those who chanced to be interested or are within hearing distance. In short, learning specifically that ten tenths make a whole results for those

* A paper read at the Arithmetic Section Meeting of the National Council of Teachers of Mathematics on June 28, 1937 at Detroit, Michigan.

whose purposes or interests happen to be focused upon that item in a general activity.

Three significant studies have been reported which seem to indicate that arithmetic cannot be taught incidentally with any great degree of success. Particular reference is made to the study made by Dr. Paul Hanna and Associates at the Lincoln School of Teachers College, Columbia University, which was reported in the Tenth Yearbook of the National Council of Teachers of Mathematics under the title, "Opportunities for the Use of Arithmetic in an Activity Program."[1] This study, as the title indicates, was a survey of the extent to which an activity curriculum contained opportunities for the acquisition of arithmetical learning. The committee concluded, "functional experiences of childhood are alone not adequate to develop arithmetic skills,"[2] and offered as a reason. "It is this very small amount of number work encountered in each grade that leads the authors to this conclusion."[3] In the summary of the study the committee made a practical recommendation in the following statement, "Until such time as the activities program is fundamentally reconstructed and a survey of these arithmetic opportunities made, a teacher will find it advantageous to approach the teaching of arithmetic through her own survey of the needs of her own pupils. If no opportunities are found for certain of the present course-of-study requirements, she will probably do the best she can to build meaning before drill."[4] It would seem that Hanna's conclusions point to a need for an arithmetic program planned in advance of teaching.

A second study, by Henry Harap and Charlotte Mapes, published in the March 1934 issue of the *Elementary School Journal*, entitled, "The Learning of Fundamentals in an Arithmetic Activity Program," was made, according to the authors, "to discover the extent to which the fundamentals are learned in an activity program."[5] The results at first glance seem to give support to the contention that arithmetic can be taught incidentally. However, a closer examination of the details of the study is revealing. For example, the authors state that, "The activities were selected deliberately because they were rich in the applications of the fundamental processes";[6] that, "The time devoted to arithmetic was the usual daily period lasting between fifty and sixty minutes";[7] and that, "All work of the pupil was kept in a notebook, which was frequently checked by the teacher. No child was permitted to leave an error uncorrected."[8] Apparently Harap and Mapes did not prove that arithmetic can be taught incidentally but rather that the acquisition of desired skills and abilities in arithmetic are obtained through the selection and direction of appropriate activities. In short, the procedure seems to have been one of planning rather definitely in advance of teaching.

Lack of space prevents a detailed discussion of a third often quoted set of articles by L. P. Benezet, Superintendent of Schools, Manchester, New Hampshire, which appeared in November and December of 1935 and in January, 1936, in the *N. E. A. Journal.* These articles have been extensively misquoted and misinterpreted. In the main, certain school officials have found in Benezet's articles a defense for eliminating arithmetic entirely in the lower grades. The facts are that Benezet might have discontinued the use of the textbook and with it the excessive emphasis on number computation but an arithmetic outline was prepared for the Manchester schools with arithmetic instruction in it for grades one and two as well as for the higher grades of the

[1] The National Council of Teachers of Mathematics' Tenth Yearbook, pp. 85–120.
[2] *Ibid*, p. 118.
[3] *Ibid*, p. 119.
[4] *Ibid*, p. 120.

[5] Harap, Henry and Mapes, Charlotte, *Elementary School Journal*, March, 1934.
[6] *Ibid*, p. 515.
[7] *Ibid*, p. 516.
[8] *Ibid*, p. 518.

elementary schools. First-hand observation of the work in the Manchester schools in the spring of 1936 leads the writer to conclude that Benezet did not prove that arithmetic can be taught incidentally either. Instead, he provided conclusive evidence that children profit greatly from an organized arithmetic program which stresses number concepts, relationships, and meaning.

The foregoing evaluation of the most significant reports dealing with incidental learning in arithmetic leads clearly to the conclusion that any course of instruction in arithmetic which produces worth while results is not an incidental but rather a planned and organized program. In certain centers this has been recognized to the extent that the courses of study for the kindergarten and the primary grades list specific number activities. Obviously those responsible for the instruction in these schools do not believe that number learning will be guaranteed unless teachers consciously provide for it. In other words, they do not chose to rely upon a hit and miss type of number learning which occurs when recognition is not given to arithmetical goals.

Although the case for an organized number program in preference to no program or to incidental teaching is a strong one, there still remains the practical question of determining the basis upon which arithmetic shall be organized for purposes of instruction. It has already been indicated that there are school systems in which number instruction begins in the kindergarten. By number instruction, however, something quite different from what is offered in most classrooms is meant. Regarding this matter Buckingham presented a helpful analysis of arithmetic in the May, 1935, issue of *Childhood Education* in a discussion of the question, "When to Begin Teaching Arithmetic."[9] In this article a distinction was made between what Buckingham termed the concrete and the abstract areas of arithmetic. Under the heading of concrete arithmetic he placed such activities as "counting objects, reproducing, matching, and identifying numbers by means of objects, solving verbal problems by actual measurement with a ruler or with pint and quart cups, reaching conclusions on the basis of one-to-one correspondence, dramatizing a quantitative situation with a consequent decision as to its meaning, putting together or adding groups of objects, taking them apart or subtracting them, playing games which require the use of number ideas, manipulating objects or pictures or number patterns for a quantitative purpose. . . ."[10] In describing abstract arithmetic Buckingham stated, "It is true that the ordinary course of study for grades one and two is heavily weighted with abstract counting and with number facts and processes."[11]

Buckingham's analysis and description of arithmetic activities contains two points of importance for this discussion. First, a clear-cut line is drawn between the sort of arithmetic which has fallen in bad repute (the abstract type) and what he terms concrete arithmetic. Secondly, Buckingham offers a description of arithmetic activities in terms of arithmetic which is precisely what Harap and Mapes did. Reference is made to such statements in Buckingham's article as, "reaching conclusions on the basis of one-to-one correspondence," "solving verbal problems by actual measurement with a ruler or with pint and quart cups," and "manipulating objects or pictures or number patterns for a quantitative purpose." From the point of view of value in the preparation of an instructional program, such descriptions certainly are clearly more specific, definite, and concrete than "buying at the store," "playing the bean bag game," "going on a trip," "preparing for Christmas," and the like, which are com-

[9] Buckingham, B. R., "When to Begin Teaching Arithmetic,"—*Childhood Education*, May, 1935, pp. 339–343.

[10] *Op. cit.*, p. 340.

[11] *Ibid.*, p. 340.

monly found in courses of study. Obviously the manner in which the goals for arithmetic teaching are expressed determines in a large measure the nature of the teaching. It would be quite impossible to attempt to achieve goals as described by Buckingham without consciously planning for them.

Fortunately there is valuable information at hand about the number interests and experiences of children in the form of number readiness inventories. An analysis of these after the pattern suggested by Buckingham would supply goals and activities for an organized number program for even the kindergarten and the pre-school. It would seem then that we should look forward to a program of arithmetic that begins when children enter school and which deals with number learning in a specific way.

The chief characteristic of the plan of instruction which has been outlined is its definiteness. Recognition is, however, given to the fact that children do learn and will continue to learn much incidentally after entering school. The act of acquiring learning incidentally is one which continues throughout life. The value of such learning cannot be minimized because it forms a large part of our mental equipment. However, there seems to be little justification for complete reliance upon incidental learning at any age or grade level when the possibilities are at hand for the development of an organized program of arithmetic adapted to the number interests and abilities of primary grade children.

Throughout this discussion the contention has been made that certain things cannot be left to chance. This has been done with the realization that much of our worth while learning seems to be obtained without much attention on our part. However, the informality of the situations in which we obtain this learning does not change the psychological fact that in the last analysis there is no such thing as incidental learning per se. In every act of learning there must be attention on the part of the learner. Certain things are seemingly learned with so little effort or attention on our part that we are at a loss to tell when or how they were learned and hence our faith in incidental learning.

If the principle is accepted that attention or interest are essential in the learning process the problem of so organizing and directing learning situations that the acquisition of the learning which is necessary for success in life will be guaranteed is a fundamental one. There are a small, very small, number of master teachers who do not need carefully outlined courses of study to direct them in guiding the attentions and interests of children to the end that worth while learning is achieved. This act involves two things. First, the interest and attentions of children must be skillfully stimulated and guided, and secondly, the teacher must have definite desirable outcomes so clearly in mind that attention is guided in the right direction. It is an art to guide the interests and attentions of children. A knowledge of the foundations of arithmetic, however, is also required to know what the possibilities of a given situation are in terms of arithmetical learning. For example, a group of children were numbering tickets from 1 to 100 for a show that they were planning. One of the pupils remarked in the course of numbering, "It is funny how I started all over again with 1, 2, 3, 4, 5, 6, 7, 8, 9 when I reached 20, 30, 40, 50, 60, 70, 80, and 90." The interests of the children had been guided toward the doing of a worth while activity but the teacher did not possess a knowledge of the foundations of arithmetic and hence with the remark, "That is very interesting," dismissed an opportunity to guide the pupils further into an insight of the decimal nature of the number system. A record of opportunities overlooked by teachers whose pupils are engaged in activity work would be enlightening.

The whole matter may be viewed from a practical standpoint. If teachers of

arithmetic possessed both the art of guiding the interests and attentions of children into worth while activities and also had the prerequisite knowledge of the foundations of arithmetic, the problem of incidental and directed learning in the area of arithmetic teaching would not be raised. Until those conditions generally exist, it seems that we are forced to agree with Paul Hanna's conclusion—"Until such time as the activities program is fundamentally reconstructed and a survey of these arithmetic opportunities made, a teacher will find it advantageous to approach the teaching of arithmetic through her own survey of the needs of her own pupils. If no opportunities are found for certain of the present course-of-study requirements, she will probably do the best she can to build meaning before drill."

Helping Mathematics with an Exhibit

By William C. Krathwohl
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This is a story of how one city is trying to overcome the prejudice which exists in some quarters against mathematics.

When the National Council of Teachers of Mathematics held its annual convention in Chicago, February 21 and 22, 1936, one of the features which attracted unusual attention was the exhibit prepared by high school students. At that time mathematics, in Chicago at least, seemed to be on its way out of the picture just as other subjects had gone before.

Someone suggested that a mathematical exhibit might not only educate and influence the public to be more tolerant toward mathematics, but might even create a demand for the subject. The question was, "Where could such an exhibit be placed?" In answer to that question, the Director of the Adler Planetarium offered space on the first floor of the Planetarium for as long a time as such an exhibit could be maintained. This was the initial impulse that set the whole project in motion. The Womens Mathematics Club of Chicago and the Mens Mathematics Club of Chicago, both exceedingly active and wide awake clubs, not only appointed a joint committee to take charge of such an exhibit, but also agreed to back the project financially. The committee is composed of Miss Laura E. Christman, Senn High School, Chairman; Miss Marie Graff, Englewood High School; Miss Frances Hubler, Tilden Technical High School; Dr. W. C. Krathwohl, Armour Institute of Technology; Miss Dorothy Martin, Bloom Township High School; Mr. Francis W. Runge, Evanston High School and Mr. Joseph J. Urbancek, Lane Technical High School.

The plan as finally evolved can be summarized as follows.

1. The object of the exhibit is to be kept constantly in mind. This object is to show the eternal and everlasting utilitarian character of mathematics both from the pragmatic as well as the aesthetic points of view. It is hoped that students will have their interests aroused to the point where they will want to study mathematics, that parents will want their children to study mathematics and that possibly parents may even take a chance and study mathematics themselves.

2. The exhibit is to contain articles actually constructed, designed or selected by students instead of by their teachers. In this way, not only will the interest of the pupils be aroused, but also the interest of the parents. It is not only the present but also the future adult population of a city that must be aroused to preserve mathematics in the schools from the grade levels up through the junior colleges.

3. Rewards are offered in the form of labels which are pasted on articles that are exhibited. By means of this device interest is kept alive long after the article has been put on display.

4. The exhibit is changed every quarter. Three months allows sufficient time in a busy city like Chicago to build up school loyalty and enthusiasm so that every member of a school can see the exhibit which is associated with his institution. During the last five months some schools have run articles in their local newspapers, and have published pictures which their own photographers have taken. In other words it is possible to make students mathematics-conscious in spite of their private opinions.

5. Two high schools chosen so as to supplement each other, exhibit simultaneously. Thus a technical high school may be paired with a general high school, a girls school with a boys school, or a suburban school with a city high school.

6. Each exhibit has some outstanding

characteristic or is built around a central theme. The characteristic or theme is chosen to attract the attention of the public, excite its curiosity, arouse its interest and make it want to see the exhibit.

As an illustration, the first exhibit made a specialty of bridges and had as a side line a model of "Job's Coffin," Interest in the San Francisco bridge which was being opened about that time, helped to stimulate interest in bridge design. "Job's Coffin," was such a curious object to be associated with mathematics, that many wondered just what such an object could be.

The second exhibit profited by the dioramas of the Century of Progress Exposition and had for its outstanding feature a diorama illustrating measurement. It showed how a person could measure the distance between two cribs in the lake and stay on dry land, instead of swimming from one crib to the other while carrying a tape measure in his mouth. Above the diorama was a drawing of it in two dimensions, and above that were two posters showing all the computation. Another feature of this exhibit was a collection of models of the thirteen Archimedian solids with such impressive and thunderous names as The Great Rhombicosidodecahedron and The Great Rhombicuboctahedron. One school newspaper stated that these were not dinosaurs or dragons but just harmless polyhedrons.

Although it may appear to be sacrilege to treat mathematical subjects so lightly, still the thought remains that possibly mathematics has lost some of its popularity because mathematicians are inclined to take themselves so seriously. If the public ever can get the idea that there is as much real joy, fun and humor in mathematics as in some of the lighter subjects, possibly they may not be so frightened when they see the word.

The following themes around which it is hoped to build some of the future exhibits illustrate how curiosity can be aroused and the public disillusioned about its fears of mathematics or its aversion to the subject:

1. *High School Mathematics Applied to Finance.*
2. *Applications of High School Mathematics to Games of Chance.*
3. *How Angles are Trisected.*
4. *How Nature Plays With Geometry (crystals, flowers).*
5. *Multiplication Without Thinking (abacus, Napier's Rods, logarithmic tables, slide rules, computing machines).*
6. *Using Mathematics to Deceive (cans that look bigger and are not, oranges that cost too much).*
7. *How Mathematics Is Used In Churches (window designs, pulpit designs, arches, Gothic constructions).*
8. *How the Designer of Parisian Gowns Uses Mathematics.*
9. *How Stout People Look Thin With Mathematics.*
10. *Who Said That Fractions Were Hard?*

Titles like some of these will make people smile. If people smile, they lower their resistance.

7. Every effort is made to give the exhibit publicity. Unless people see the exhibit, it might just as well not be there. Reporters from the daily papers are beguiled into visiting the exhibit. Students are invited to take pictures of it. If they happen to pose some fair object with the rest of the collection, there is no objection. Association does wonders. Principals clubs and associations are informed discreetly that a school of one of their members has an unusually attractive exhibit. In short the committee does not believe in hiding its light under a bushel.

What the outcome of this campaign is, remains to be seen. It does not seem possible that so much effort could be expended on such a worthy cause without a happy ending. At least here is one suggestion which may be of help to schools in other communities.

The Concept of Dependence in the Teaching of Plane Geometry

By F. L. Wren
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The concept of the interdependence of geometrical magnitudes is by no means new. Although instances of mathematical dependence are to be found scattered throughout the mathematical works of the early Greeks and Egyptians, it was Descartes who formulated the principle that served to crystallize the concept of dependence, or functionality, into one of the most significant of all mathematical concepts. In his new approach to geometry, published in 1637, Descartes introduced a frame of reference consisting of two mutually perpendicular lines as axes and their point of intersection as the point of reference. All points of the plane were then located by means of their respective distances from these axes of reference, a suitable notation for distinguishing between constant and variable distances was set up, and a curve was defined as the locus of a point moving in such a way that certain specified relations between its distances from the respective axes were preserved. Through such a transmutation in geometric technique Descartes was able to definitely systematize the method of applying algebra to the effective study of geometry. Though it was Descartes who devised the technique that has proved so efficient as a key to the study of functional relations, the contributions and extensions made by Leibniz and Newton must not be overlooked. It was, in fact, Leibniz who introduced the word *function* to designate the magnitudes involved in Descartes' concept of curves generated by a moving point.

Through the researches of Descartes, Leibniz, Newton and others the concept of functional dependence was definitely established as a concept of fundamental mathematical significance. It is rather strange that teachers of mathematics have been so slow in recognizing its importance as an instructional medium. Most of the emphasis in the study of functionality on the secondary level has been confined to the analysis of interdependent variation by means of algebraic techniques. In plane geometry this has resulted in a tendency to overlook the fact that there are many aspects of dependence which are not expressible in the conciseness of an algebraic formula but which are deeply significant to a fundamental understanding and appreciation of the real nature of geometrical subject matter.

In 1872 Felix Klein, in outlining his famous Erlangen program, gave the following definition of geometry: "Geometry is the study of the invariants of a configuration under a group of transformations." Thus the structure of geometry may be outlined as follows:

1. Select the space, that is, determine whether the geometry to be studied is in one, two, three or higher dimensions.
2. Select the element, that is, specify the undefined elements. In ordinary three-dimensional geometry the undefined element is generally taken to be either the point or the straight line. This, however, is not necessary.
3. Build configurations such as triangles, quadrilaterals, circles, polygons, etc.
4. Select transformations. In elementary geometry, the two most frequently used groups of transformations are those of motion and projection.
5. Study invariants. An invariant of a geometric configuration is a property

that does not change in the process of being transformed. For example, length of a line does not change when the line is moved about in space but it does change if the line is projected from one plane to another, hence length is an invariant under motion but not under projection.

From the above outline of the structure of geometry it is evident that the very nature of the geometry to be studied is dependent upon making certain basic choices. In the geometry of the secondary school the space in which we are interested is either two-dimensional (plane) or three-dimensional (solid). The undefined element is the point and the transformations are those of rigid motion, namely, rotation and translation. Under these transformations such geometric properties as length, distance, size of an angle, united position of point and line, area, etc., are invariant properties. We proceed, then, to analyze the geometric configurations in terms of these invariants, for example:

1. *Two triangles are congruent when their sides are of the same respective lengths.*
2. *Two triangles are congruent when two pairs of sides of the same respective lengths include angles of the same size.*
3. *A circle is the locus of a point moving at a given distance from a fixed point.*
4. *Two circles are congruent if their radii have the same length.*

Although the point may be taken as the undefined element of the geometry of the secondary school and all other elements defined in terms of it, from a pedagogical point of view, this is very undesirable. It is not good psychology to crowd the young mind with so many formal definitions. No significant mathematical rigor is lost in taking for undefined elements in secondary geometry such terms as point, line, surface, plane, solid and space. An intelligent comprehension of such concepts can be established intuitively and it is pedagogically unsound to attempt to build up definitions of these concepts that could be accepted as technically correct.

Any system of geometrical thought is essentially logical in nature and consequently is dependent upon a set of fundamental assumptions and definitions. It is thus that we have Euclidean and non-Euclidean geometries. Euclidean geometry, the geometry of our ordinary experience, is based upon a system of postulates including Euclid's famous fifth postulate: "Through a point there can be one and only one line drawn parallel to a given line." Non-Euclidean geometries are built upon systems of postulates which are identical with the Euclidean system except for a contradiction of the fifth postulate. The different systems are consistent and constitute geometries which are logically sound and their conclusions and interpretations are significantly different from those of Euclidean geometry only when the fifth postulate is involved. This dependence upon fundamental assumptions is a significant characteristic not only of a complete system of geometrical thought, but also of the argument involved in the demonstration of individual theorems. Why should the student not have the opportunity to evaluate the consistency and reasonableness of hypotheses and to weigh the significance of conclusions in terms of the hypotheses upon which they are based? Why should they not be cognizant of the chain of deductions leading from hypotheses to conclusions? Why should they not learn from their geometry the significance of implication and the techniques of demonstration?

In any geometric configuration there exist intrinsic interrelations among the constituent elements. An analysis of this interdependence of elements is one of the most effective techniques for discovering the characteristic properties of the configuration and portraying its complete geometric significance. For example:

1. *The area of a triangle depends upon the lengths of the altitude and base.*

What happens to the area when either the altitude or the base is doubled? What happens when both are doubled?

2. *How do the circumference and area of the circle depend upon the radius? Which is affected more by a change in the length of the radius?*
3. *In a cylinder $V=\pi r^2 h$. Which would increase the volume more, to double h or to double r?*
4. *In a given triangle, how is a side affected by increasing the opposite angle?*

Questions such as these help to bring to light the exact nature of any geometric configuration under investigation. Critical analysis and intelligent interpretation of such configurational dependence will contribute to enriched geometrical comprehension.

The study of geometrical dependence is further enhanced by the principle of continuity which asserts that a proposition which has been established in relation to a given figure will remain true when that figure changes continuously subject to the conditions controlling its initial construction. The interrelated concepts of dependence and continuity unite to replace a static mechanical treatment of geometrical subject matter by a dynamic functional program of instruction. It behooves every teacher of geometry to utilized the full benefits of such an approach to the study of geometrical subject matter. As an illustration of the full significance of the introduction of these dynamic concepts into the teaching of plane geometry, consider the implications as to related thinking embodied in the two following theorems:

1. *The angle included between two lines of unlimited length which meet a circle is measured by one-half the sum of the intercepted arcs.*
2. *In a triangle the square of the side opposite a given angle is equal to the sum of the squares of the other two sides diminished by twice the product of one of these sides by its projection upon the other.*

It is true that the concept of directed line lengths must be introduced in the consideration of the above theorems for their full significance. Why should we not use the concept of directed line lengths in the teaching of plane geometry as a significant aspect of the concept of directed numbers introduced in the algebra of the junior high school? The consideration of other similar groupings of significant theorems will enhance the value of these dynamic concepts of dependence and continuity as instructional mediums in plane geometry.

In Figure 1 it is to be noted that any angle inscribed in the arc BAC will be equal to angle A. Hence it evident that the elements, a, A, R (radius of circumscribed circle) are dependent elements; i.e. given any two of them the third is determined.

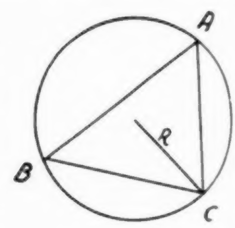

FIG. 1

In the triangle ABC (Fig. 2) let h_b and h_c represent the altitudes upon the sides b and c respectively. Let CA be extended to D so that $AD=AB$, then $CD=b+c$. Draw $DE\perp$ to CF

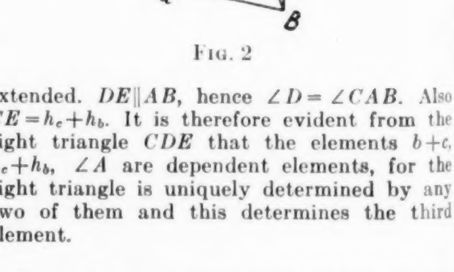

FIG. 2

extended. $DE\parallel AB$, hence $\angle D=\angle CAB$. Also $CE=h_c+h_b$. It is therefore evident from the right triangle CDE that the elements $b+c$, h_c+h_b, $\angle A$ are dependent elements, for the right triangle is uniquely determined by any two of them and this determines the third element.

Since it takes only two points to determine a line, we say that three points are *dependent* if they are on the same line. This concept of dependence has some very

interesting and important applications to the construction of geometric figures. A triangle is uniquely determined by three points not on the same line. In more general terminology this statement would read: *A triangle is uniquely determined by three independent points*, or still more generally, *a triangle is uniquely determined by three independent elements (or conditions)*. The truth of this last statement is illustrated by the congruency theorems which require three independent elements.

Since the sum of the angles of a triangle is always 180°, the three angles are dependent elements. Why are two triangles whose three angles are respectively equal not necessarily congruent? Why do three lines through the same point not determine a triangle? Two other less evident illustrations of dependent elements are to be seen in Figures 1 and 2.

Using both direct and indirect elements related to the unique determination of triangles, three sets of three dependent elements each have been exhibited. They are

I. A, B, C
II. a, A, r
III. $b+c, h_c+h_b, A$

Such sets of dependent elements are indispensable aids in an effective approach to exercises in the construction of triangles, It is the purpose of this discussion to call attention to an aspect of this concept of dependence that seems to be generally overlooked in the teaching of plane geometry. To simplify the discussion, two specific examples will be developed in the hope that inferences will lead to the individual development of many others.

A simple construction problem that frequently comes early in the geometry course is:

To construct a triangle having given two angles and the included side.

Let us specify that the given elements are B, C, a. After this construction has been completed additional construction problems may be derived from it by using the dependent elements of sets I and II. These problems may be used as instructional aids, supplementary drill material, or as material for enriching the study of construction. It would indeed be fine instructional technique to have the students derive and construct such supplementary problems. From set I it is evident that, once angles B and C are known, angle A is also known. Hence the given elements imply the following elements as given:

1. A, B, a 2. A, C, a.

From set II it is evident that, since angle A and side a are known, R, the radius of the circumscribed circle, is also known so that we have:

2. A, B, R 4. A, C, R 6. $B, C, R°$
3. R, B, a 5. R, C, a

Another interpretation of set I is that if we have given one angle of a triangle we also know the sum of the other two. Applying this to problems 3 and 5 above we obtain

7. $R, A+C, a$ 8. $R, A+B, a$

Hence from the original set of elements there have been derived 8 additional sets of given elements with which triangles may be constructed. Each of these problems probably have individual constructions which are independent of the original problem, B, C, a. It is quite evident, however, that all of them may be reduced to the original problem.

Another illustration of interest is to be found in the following construction problem which would be placed somewhat later in the instructional program than the one above.

Given the elements $b+c$, h_c+h_b, a, to construct a triangle.

After the construction has been completed an application of the dependent elements of set III will replace the given elements by

1. $b+c, A, a$ 2. h_c+h_b, A, a.

An application of set II to each of these derived sets produces

3. $b+c, A, R,$ 5. h_c+h_b, A, R
4. $b+c, R, a$ 6. $h_c+h_b, R, a.$

A combination of set I with sets 1, 2, 3, and 4 above will give

7. $b+c, B+C, a$ 9. $h_c+h_b, B+C, a$
8. $b+c, B+C, R$ 10. $h_c+h_b, B+C, R.$

If it is furthermore recalled that the perimeter ($2p$) is the sum of the sides, that is

$$2p=a+b+c$$

the following twelve sets may be derived:

11. $2p, A, a$ 17. $2p, b+c, A$
12. $2p, R, a$ 18. $2p, a, B+C$
13. $2p, A, R$ 19. $2p, h_c+h_b, a$
14. $2p, b+c\ R$ 20. $2p, h_c+h_b, A$
15. $2p, B+C, a$ 21. $2p, h_c+h_b, B+C$
16. $2p, B+C, b+c$ 22. $2p, h_c+h_b, b+c$

Thus twenty-two additional construction problems have been derived from the original one through successive applications of the concept of geometric dependence.

Similar possibilities exist in all construction situations in geometry. Such instructional technique not only provides the teacher with an enriched program of geometric teaching but also affords the student the opportunity for a more significant understanding of the real nature of geometric construction.

Geometry is not a dead subject although it has for a long time languished in an educational coma induced by a decidedly biased selection and organization of subject matter as well as by a seriously apathetic type of instruction and motivation of learning. Instead, geometry is pulsating with great potentialities as a functional school subject. By no means of least importance in an instructional program designed to realize some of the really significant values of geometry as a school subject is the use of the interrelated concepts of dependence and continuity to replace a static mechanical treatment of geometrical subject matter by a dynamic functional program of instruction. Teachers of geometry must rally as a group to the challenge to depart from traditional dictates and strive to effect a more educationally significant arrangement of geometrical subject matter while, at the same time, they must individually resolve to avail themselves of every opportunity to make their instructional program one that is dynamic in its creative possibilities, substantial in its subject content and functional in its attention to environmental opportunities.

What Shall We do with Our Unfit?

By JOSEPH SEIDLIN
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ALL is not well with the status of mathematics in our schools. Teachers of mathematics more than ever before are under fire on a great many fronts. In particular let me mention a few of our more or less worthy opponents: boards of education, the pure and simple educationist, the left-wingers of the progressive school, the administrative officers of our local school systems, the state and large city superintendents or commissioners of education, the "man in the street," the university professors of mathematics. The criticisms levelled at the teaching of mathematics are in the main familiar to all of us. I shall mention a few of the more specific charges against us partly in review and partly as a basis for my talk.

From a board of education:

"Isn't it strange that so many of us on the board feel that we ourselves gained absolutely nothing from the courses in algebra and geometry we were required to take in high school. We don't deny the value of mathematics in scientific fields, but in the education of the average boy or girl it must not be permitted to occupy too much space."

From the educationist:

" . . . Times and seasons alter. It would seem that with changing ideals must come changes in the aims of education and some changes at least in the methods of teaching. But I am wrong and to prove that I am wrong all I need to do is take you into most of our *recitation* rooms in that holiest of subject matter—mathematics."

From the university professors of mathematics:

(a) Our high school teachers of mathematics think it a waste of time to study mathematics.

(b) In our high schools bad teaching and bad learning are a consequence in large part of the ignorance of teachers.

(c) Of all subjects ours (mathematics) as taught, is most pedantic. This, I believe, is due to the fact that, contrary to general opinion, mathematics is a refuge for an inferior type of teacher.

(d) " . . . the formal, manipulative, exercise-working teacher of mathematics is, in my opinion, a much greater menace to mathematics, than. . . ."

(e) " . . . and my impression of the teaching profession (in mathematics) . . . is that they are a bad lot. Therefore . . . , in my opinion, the study of mathematics in high schools should be absolutely prohibited."

From a commissioner of education:

"We are eliminating mathematics because we do not know how to eliminate the poor teaching of mathematics."

I suppose our combined blood pressure at the moment is very high. Mine is, despite the analysis that is to follow. In this connection I used to get some comfort out of a statement Mr. Betz made a few years ago. He said, "The arguments against mathematics come in the main from people who are unacquainted with the newer objectives and methods in the teaching of mathematics." I fear it is more serious than that. What hurts the cause of mathematics is not so much that our critics are unaware of the newer objectives and methods but that the majority of teachers are.

When we hear some of these so severe criticisms of the teaching of mathematics, criticisms so harsh that they hurt, our reaction—and a natural reaction it is—is porcupinish. We remonstrate, and truthfully, by a show of indignation. "Why, the idea, I don't teach geometry by assigning four pages of axioms and postulates and definitions as a lesson; I don't begin the teaching of algebra by two weeks of deadly substitution; I don't teach mathematics

that way. Certainly *you* don't. But it is being taught *just that way* in hundreds of our schools.

You know that when Dr. Reeve is given the opportunity to speak, irrespective of his topic, sooner or later, he makes an impassioned plea for the support of the National Council. On one such occasion he mentioned two fairly large groups of teachers who do not support the Council. I quote: "The first group consists of those teachers who, according to their own testimony, have never heard of the N. C. T. M. or *The Mathematics Teacher,* to say nothing of the yearbooks. . . . In the second place, many teachers of mathematics who know about the existence of the Council, the magazine and the yearbooks do not take enough interest in the Council's welfare to join the organization or to keep up their membership once they have joined."

Perhaps, in that statement, Dr. Reeve implies that the real seriousness of the situation for all of us is only partly this ignorance of the work of the Council or the non-support of the Council. In the main, the danger lies and has lain for a long time in the fact that this relatively large group, in numbers the predominant group, is ignorant of and uninterested in mathematics and the teaching of mathematics. The only reason they hold on to their jobs is physical self-preservation and a natural or acquired rut-phobia.

And so I raise the question "What Shall We Do With Our Unfit?" I want to make it clear that my emphasis is a good deal on "We." What Shall *We* Do. . . . For as I see it, if *we* don't do anything about them some other groups will. And when they do, as they have been doing, the elimination is ruthless and harmful to our growing boys and girls and to our ideal of democracy. It is like eliminating the drinking of water because some streams are polluted. For whether we like it or not, water is a physiological necessity. Let us, instead, purify our streams.

I am reasonably convinced that we have a fairly representative number of capable, energetic, missionary teachers of mathematics. Also, it seems to me, that under a succession of capable leaders this group is fairly well organized.

Don't let us fight the social studies group. Don't let us fight the general educationist save on occasions when he becomes a nuisance or a menace to the practical business of educating youngsters. Don't let us fight the superintendents of schools or even the college professors of mathematics. Rather, let us pool all our abilities and energies toward setting our own house in order. The simplest way is strictly forbidden by law. We might claim justified or justifiable homicide but I fear we couldn't get away with it. Not that we need to overlook our natural allies—old age and death.

But our main job is creating and, wherever humanly possible, recreating teachers. I am not, at this moment, accusing our teacher-training agencies of incompetency or neglect. Nor am I completely absolving them of all blame. But that is a different story. My present concern is more practical and more immediate. As President Roosevelt insisted in one of his "fireside chats": What can be done or what must be done *NOW?* More specifically, what can this group, what can the N. C. T. M. do to help eliminate the unbearably poor teaching of mathematics in our schools?

Are there enough of us, are we evenly enough spread, to observe, assist, and report every teacher of mathematics in the country? What effect would the mere launching of such a scheme have upon thousands of semi-conscious, lethargic teachers? As I see it these clock-punchers, these chore-performers are our most serious problem. It may be quite impossible to infuse any life or ambition into a great many of these, but certainly a concerted and sustained interest in them may generate some interest for their own work. For a good many of them perhaps that is all that can be done; for some it may be the beginning of "better teaching." Whatever success we may hope for, it is a huge task.

A good many of our younger teachers,

live teachers, are perplexed. They are bewildered by the multiplicity and complexity of activities into which they are drawn and become disintegrated or from which they shy away and become "subject-matterists." Paraphrasing one of our philosophers,—many potentially good young teachers have become a prey to propaganda from *progress* and *up-to-dateness*. Whatever may be the strength numerically or "politically" of the so-called left-wingers of the Progressive School, they do make a lot of noise and they do achieve a lot of prominence in print. To appease them our friends the centrists often say some strange things.

Now it seems to me that the faculties of the departments of the teaching of mathematics whether imbedded in schools of education or adjacent to departments of education in liberal arts colleges must take a gentle but firm stand against educational sentiments insulated by deceptively logical ideas.

And, finally, what about the new crop of teachers we dump on the market year by year? We certify, qualify, encourage too many young men and women of mediocre or inferior qualifications to enter or continue in the field of teaching. Aside from the purely selfish motive of expansiveness to which departments of education, schools of education, and Teachers Colleges are unfortunately addicted, why do we permit, nay give our blessing to, definitely inferior personalities to further clutter up the all ready crowded field of teaching? Do we need the cataclysm of a depression or the dénouement of a recession to help us guide misguided youth into fields other than teaching? It may hurt our pride to admit that we are not magicians and we would have to be at least that to make teachers out of the clay put in our hands.

My face was red, as yours must have been, when the Carnegie Foundation Study in Teacher Training was published. I refer particularly to that part which . . . let me quote: " . . . students in teacher-training schools and in university departments of education are substantially lower than comparable liberal arts students in the scores which they secure on nearly all tests, including tests of intelligence, English, mathematics, foreign languages, natural science, and social science."

If there is any sense in the well-known platitude that "a chain is as strong as its weakest link" let us pay a bit more attention to our weak links. I hope that nothing I have said has given anyone the impression that this group, including myself, "good as we are," are good enough. When anyone begins to feel that he is good enough he isn't. But the problem that looms large to me, perhaps because we have paid so little attention to it, is not, for the moment, how to make good teachers better, but how to make poor teachers less devastatingly poor.

Are we men and women or mice of the corresponding sex? Are we educators with a great social responsibility toward our school boys and girls, or are we, as some call us, the champions of vested interests, the Lord Protectors of our weak sisters and brothers? Shall we ally ourselves with administrators who are courageous enough to eliminate from their staffs any instructor who not only IS *but is content to remain* an uneducated person? What constitutes a successful operation for the removal of cancerous growth? What apologies do surgeons offer for "cutting in deep?" How shall we go about affecting a clean cut separation of teachers of mathematics from employees in charge of classroom recitations in (so-called) mathematics?

I have diagnosed and stated as clearly as I know how one of our major ills. I have suggested that this group, that the N. C. T. M. must affect at least a partial cure. I appreciate the magnitude of the task. I realize, too, that it is a "delicate" problem. I insist, however, that what is done must be done *now* and preferably by this group.

◆ THE ART OF TEACHING ◆

Inverted Geometry

By Daniel Luzon Morris
The Putney School, Putney, Vt.

After a person has passed through the "discipline" of plane geometry, solid geometry, and trigonometry, he sometimes realizes the transcendent beauty of plane geometry. Why should he not realize this while he is learning it?

Geometry starts with definitions of points, lines, angles, and so forth. The whole of plane geometry is concerned with nothing but points and lines. But points and lines are abstractions. The definition of a line is that it is an extension. It has length, but neither breadth nor depth. A point has not even extension. The student is taught, as soon as he meets points, that a true point cannot be seen, because it has no thickness in any direction.

The naive imagination can immediately conceive of a solid object, because the naive imagination lives in a world of solid objects. Solid objects can be seen and felt. They are real. A plane, as something with length and breadth but no thickness, is abstract; but a plane, as a boundary of a solid, has a very real existence. When I bump my head on a door, my head is meeting the plane bounding the door. That is real.

It would seem therefore that if geometry could be taught by starting with real things—solids and planes—and from them proceeding to the abstractions—lines and points—(and we shall see as we proceed that even these need not be so abstract) it would immediately be acceptable to the adolescent mind. Throughout plane geometry, as usually taught, if a pupil asks, "What is the use of this particular proposition?" The teacher must reply, "This proposition will be necessary in the proof of those coming later on in the book." Throughout plane geometry it seems to the student that he is going through the most frightful contortions to prove things that should be evident to a new-born babe. "If two lines are perpendicular to the same line they are parallel." The old plaint is so reasonable: "I don't have to prove it, I know it."

Descriptive geometry is a bugbear to most students who get to it. It is so complicated! Yet the other day I proved a most abstruse problem in plane geometry to a child by the use of descriptive geometry—or at least the principles of descriptive geometry—and the child immediately saw the applicability and accuracy of the proof. I should have hated to try to demonstrate that same proposition to a college student.

I should like to teach geometry to children in a way exactly the opposite from that usually used. I should start with descriptive geometry, follow that—or combine it—with solid geometry—and conclude with plane geometry as the exquisite culmination of geometrical thought.

I think that I should start by having the students make a regular tetrahedron. This is done by constructing four equilateral triangles on a sheet of paper (there we have Euclid's first proposition—but we don't mention Euclid yet—the matter is obvious—doesn't need to be proved). The figure is cut out with scissors, and folded on the lines. The edges are glued together. Kindergarten play? Perhaps. But at this point I should exhibit one of my own models, say that of the inter-

section of two octahedra, also cut out of paper with scissors and glued together at the edges.

Now, having learned how to do it, we make models of, say, a cube and an octahedron. The octahedron is a little more complicated, and a little harder to plan out. Also, if the compasses happen to slip in the middle of the construction, the thing won't fold up right. Next we should construct a roof; that is to say, a triangular prism with the bases perpendicular to the lateral edges. But it would be a roof, not a prism.

Now descriptive geometry begins.

Let's draw a picture of the roof—but an accurate picture, so that from it any one else could construct a roof exactly similar to ours. That's what an architect has to do after he has planned a house. First we can put the roof on a sheet of paper and trace a line around it. That gives its outline. Now, how can we draw the ridgepole in the right place? Suppose we flop out one of the ends of the roof, folding it down along its trace on the paper. Next draw a line around this. Do the same thing at the other end. Connect the two turned down peaks, and we have the ridgepole in the right place. We don't have to prove it, we can see it.

Now, if we simply have the drawing of the house from on top, how is the next person to know how high the ridgepole is? Well, let's leave one of the ends that we turned down in the drawing for him to work with.

Here we have what the descriptive geometer would call a problem in the intersection of two planes, each plane being determined by its trace on the plane of projection and its angle of inclination. In beginning descriptive geometry, this is a problem of moderate difficulty. In building your own paper house every step is simple.

The further development is the natural one. We continue to use one plane of projection as long as it seems to be the simplest way to get our results. When the drawings begin to get a little too complicated, with the myriad of "indication lines" that are necessary in this type of projection, we find ourselves forced to use two planes of projection at right angles to one another. But nothing that has been learned is lost effort. We shall continue to find it necessary to determine angles of inclination, even if they are not given in the statement of the problem. We shall continually be turning plane figures down into one plane of projection to measure them. And we shall be making more and more complicated models.

By the time that the first few models are completed it will be evident to anyone that two planes intersect in a straight line. Of course that's true, why make a fuss about it? But then we find that two straight lines meet in a point, and three planes meet in a point. Now a line is no abstraction—it is one of those edges that we are gluing together—a mathematically correct as well as naively obvious conception. A point is real—it may be sharp moreover.

The whole of solid geometry unfolds like our models.

Then we come to the climax of the course—plane geometry.

We have already seen that two triangles are equal if their respective sides are equal—at least it looks and works that way. If they aren't, they're pretty close. Well, let's see if they aren't *exactly* equal, and why. Don't forget that by cutting up a square composed of sixty-four squares, and putting it together in another way, you can make a rectangle having sixty-five of the same size squares in it. You can't believe everything you see.

And so the student attacks plane geometry in an entirely new spirit. It has justification. And I believe that plane geometry may become, to these children the second time through, the fascinating study it is to adults.

* * *

The foregoing was written some two years ago, while I was engaged in bio-

chemical research. At that time it seemed very unlikely that I would ever have a chance to put the ideas mentioned here into practice.

As it chanced, I started teaching mathematics shortly thereafter, at the Putney School, in Putney, Vermont, and I have had a chance to see if these frankly Utopian notions could in any way be put into practice. I say Utopian, for the whole course as outlined here can of course not be feasible under the present college board examination system.

The answer is, that at least in part it can be done. My plane geometry course starts out with three to five weeks of the descriptive geometry in one plane mentioned at the start of this outline. This is followed by a week or ten days of elementary logic, based largely on Lewis Carroll's *Symbolic Logic;* and then the usual propositions of plane geometry are taken up. The students experience very little difficulty with the descriptive geometry, seem to enjoy it tremendously, and enter the plane geometry with an enthusiasm that enables them easily to make up for the time "lost." As a result of this, there is no sacrifice of the work required for the college board examinations.

Later, in conjunction with the solid geometry course which I happened to be teaching at the time, more complicated problems in descriptive geometry are taken up, and a thorough study of the subject is possible.

IN OTHER PERIODICALS

By Nathan Lazar
Alexander Hamilton High School, Brooklyn, New York

1. Bush, L. E. "The Introduction of Invariant Theory into Elementary Analytic Geometry." *National Mathematics Magazine.*
   (a) 12: 82–89. November, 1937.
   (b) 12: 131–137. December, 1937.

"Because of the manner in which this interesting topic is presented in the usual text book, the student is often left ignorant of the power and beauty of the method of invariants." The author presents another approach which, he believes, is free from the defects he pointed out. No knowledge of mathematics is assumed beyond that usually given in a first course in college algebra.

2. Cowley, E. B. "Ratio and Proportion in High School Curriculums." *School Science and Mathematics.* 37: 1079–1088. December, 1937.

The purpose of the article is to study the high school pupils' grasp of relationship of measurable quantities as revealed by the comprehension of ratio and proportion. It consists of four parts:

(a) An examination of textbooks in high school mathematics to ascertain whether the treatment given to ratio and proportion brings out the idea of relationship.
(b) A similar study of textbooks in other subjects—especially chemistry and physics.
(c) An analysis of some published articles written by teachers of chemistry and physics regarding the amount and character of mathematics required by high school pupils studying these subjects.
(d) Results obtained from a questionnaire sent to pupils studying chemistry or physics in the public high schools of a large city.

The following are some of the author's conclusions:

(a) "The concept of relationship is slowly but surely becoming the dominant idea in the treatment of ratio and proportion in text books in mathematics, chemistry, and physics. There are a few conspicuous exceptions.
(b) "Published articles written by teachers of chemistry and physics indicate that these teachers differ widely in their use of relationship in proportion.
(c) "Questionnaires answered by pupils studying chemistry or physics show (1) that from 70% to 80% of the pupils have a sufficient grasp of the idea of relationship to answer easy questions expressed in terms of everyday affairs, and (2) that some of the pupils have difficulties with the problems because they look at proportion as merely a mechanical device worked by arbitrary 'rules.'
(d) "The importance of the idea of relationship inherent in ratio and proportion must be emphatically presented to teachers, writers of textbooks, supervisors, administrators, educational diagnosticians and curriculum builders."

3. Curtiss, D. R. "Fashions in Mathematics." *The American Mathematical Monthly.* 44: 559–566. November, 1937.

The retiring presidential address delivered at the meeting of the Mathematical Association of America at Pennsylvania State College, September 7, 1937.

It has often been noted by many writers that there are fashions in mathematics as there are in clothes and in literature. No one has, however, recorded how the volume of papers, distinguished and undistinguished, in a special field has increased, reached its peak, and then fallen off. In this article the writer reports the results of a study made in accordance with the program just mentioned. The technique used and the pitfalls inherent in it are also indicated.

The cycles of interest in various mathematical topics resemble a gold rush. "Someone strikes gold, and the rush is on until the diggings become less productive; then on to the next rich prospect."

The author closes with the following remarks:

"Let me conclude with another figure of speech, perhaps unfortunately borrowed from military science, but in some ways more apt than the comparison I have made to a gold rush. I think of mathematicians as behind lines that

separate them from the territory not yet acquired. Some genius marks out a great salient to be added to the conquered regions. Other mathematicians occupy it, and push it out still further. Finally it merges with other salients, which tend to disappear as such and new wedges are extended from the consolidated territory into regions previously unconquered. These salients correspond to what I have called fashions. They are the landmarks of progress."

4. Escott, E. B. "Rapid Method for Extracting a Square Root." *The American Mathematical Monthly*. 44: 644–647. December, 1937.

The writer claims the method he describes for extracting square root is "probably the most rapid method yet discovered. As it gives the square root in the form of an infinite product, it is especially well adapted for use with a computing machine. In computing a table of square roots by the method of differences it is important to have an independent method of computing an occasional value and this method is very good for that purpose."

5. Higgins, T. J. "Slide-rule Solutions of Quadratic and Cubic Equations." *The American Mathematical Monthly*. 44: 646–647. December, 1937.

A method is described for determining the real roots of quadratic and cubic equations from one or two settings of an ordinary slide-rule. The author believes that "there is no American publication that contains this information. Yet an understanding of such solutions would be of real value to those who must frequently solve such equations, and to whom quickness, accuracy, and release from laborious computations are prime considerations. In particular, students of engineering, the natural sciences and those engaged in statistical and actuarial studies find these solutions especially useful. Not only are they independent modes of obtaining roots, but they may be used to check the solutions obtained by the use of Horner's Method, Newton's Method, the usual formulas, or other conventional means."

6. James, Glenn. "Number and the Four Fundamental Operations of Arithmetic." *School Science and Mathematics*. 37: 1025–1028. December, 1937.

The writer complains that "in a recent class of sixty, with ability above the average, containing teachers from the first grade to junior college and freshmen to graduate students," he found not a single student who clearly understood the concepts of number, addition, subtraction, multiplication, and division. He attributes this condition to the tendency to teach arithmetic as a rule of thumb. In order to aid in this matter the writer attempts to state briefly the meaning of the above mentioned concepts, "negate certain very prevalent misconceptions of them, and show how clarity on these points carries students over some of the hardest places in arithmetic and algebra."

7. Jelitai, József. "The Fourth International History of Sciences Congress." *National Mathematics Magazine*. 12: 77–81. November, 1937.

A concise but illuminating report of the proceedings of the Congress that was held in Prague on September 22–27, 1937.

8. Kempner, Aubrey. "The Mathematical Association and Mathematics in the Secondary School System." *The American Mathematical Monthly*. 44: 634–637. December, 1937.

This article is another welcome sign that the teachers of mathematics in the colleges and in the universities are at last awakening to the realization that they are partly to blame for the precarious situation of mathematics throughout this country. "It seems to me that, up to a few years ago, a quite unreasonable amount of distrust and suspicion existed between university people and high school people. Common pressure in a common cause is beginning to drive us together. We look at each other, talk to each other, listen to each other, and discover that we really speak the same language. The National Council of Teachers of Mathematics has approached the Association concerning the possibility of a joint session, and the Association sincerely hopes that such a meeting may be arranged."

As the readers of *The Mathematics Teacher* are undoubtedly aware the Joint Commission on the Place of Mathematics in the Secondary Schools has already been formed, and that a preliminary partial report will be issued in the near future.

9. Steen, F. H. "A Method for the Solution of Polynomial Equations." *The American Mathematical Monthly*. 44: 637–644. December, 1937.

"This paper presents a method of finding a fractional approximation to a real root of a rational integral equation with numerical coefficients. The method usually requires less computation than the customary processes for 'extraction' of square root and cube root, and less than 'Horner's Method' of solving equations, and is better adapted to the use of computing machines. It is of particular value when a high

degree of accuracy is required in the approximation."

10. Wexler, Charles. "The Major in Mathematics." *The American Mathematical Monthly*. 44: 586. November, 1937.

The author urges "that a mathematics major should be defined, not by the number of units in mathematics, but by the subject matter studied; and that this subject matter should include not only a course in advanced calculus (making at least three semesters in calculus or the equivalent required) but also a one or two semester course in the theory of functions of a complex variable." He also points out the defects of the prevailing requirements and the advantages to be derived from the acceptance of his proposal.

NEW BOOKS

Mathematics in Certain Elementary Social Studies. By Eugene W. Hellmich, Bureau of Publications, Teachers College, Columbia University, 1937. 125 p. Price, $1.85.

Among the major educational trends during recent years is the increased emphasis given to the social studies. While no thoughtful person would question the values to be derived from this work it has become increasingly clear that the problem of realizing these values is not alone the problem of the social studies teacher. There is, in fact, no teacher of the secondary school which does not share this responsibility and that of the mathematics teacher is by no means the least for the literature of the social studies cannot be fully understood by any reader who is unfamiliar with the language of mathematics. Dr. Hellmich has recognized this problem and through a careful and thorough examination of representative social studies materials from the junior high school level, from prescribed introductory courses in college and from elementary economics has focussed attention on those "mathematical concepts, facts and processes" with which the pupil should be familiar if he is to read the literature intelligently.

He defines mathematics as including "those words, figures, and other symbols that have been adapted to describe the results of counting and measuring and the methods and language that have been developed in the course of manipulating, comparing and presenting the results and processes involved in counting and measuring" and examines the selected materials for mathematical content consistent with this definition. The "concepts, facts and processes" thus determined are presented in tabular form with a helpful analysis of their implications for teachers of mathematics. Dr. Hellmich has recognized, however, that while it is important to know what these "concepts, facts and processes" really are it is just as important to know the "connections" in which they occur in the social studies if meanings and appreciations are to be emphasized rather than factual knowledge only. He has therefore included in his study many tables as well as actual illustrations showing the nature of these connections and it seems to the reviewer that this is one very strong feature of this work for it provides an opportunity to associate the learning of mathematics with situations which reveal its possibilities as an instrument of social progress.

Having drawn attention to the mathematics that is actually needed for a comprehensive understanding of the social studies materials in the junior high school, Dr. Hellmich discusses the extent to which provision is made for the learning of this needed mathematics. After a careful examination of twenty-seven mathematics textbooks, eighteen of which are used in the elementary school and nine of which are in wide use in the junior high school, he states that "the mathematical activities required in the social studies, aside from the construction of graphs, are adequately provided for in the arithmetic textbooks" and recommends that "provision be made for a consistently earlier introduction of graphs."

After studying "prescribed introductory courses in social studies at the college level" Dr. Hellmich makes the following significant statement "The social studies will use mathematics to an even greater advantage when there is more recognition of the interrelationship between mathematics and the social studies and when teachers of mathematics teach more *about* mathematics in the teaching of mathematics.... The discussions in the social studies are concerned not only with the nature of mathematics as a basis for describing its place in the evolution of society but also with its place in the development of thought." He then gives numerous suggestions as to the interrelation of mathematics and social studies and directs attention to certain limitations in our present secondary school program. He finds the mathematical demands in the study of elementary economics "considerably more extensive than in the more general social studies materials" and points out that the present mathematics program in the usual secondary school fails to meet these demands.

In view of the increasing emphasis given to the social studies this is a timely and suggestive contribution. It should be most effective in helping both the mathematics and social studies teachers to better understand their mutual problems and throws considerable light on the interrelation of their respective areas. Of particular importance to the social studies teacher is Dr. Hellmich's emphasis on mathematical method and the fact that in certain social studies materials examined "the method of reaching conclusions in geometry is shown as the method of reaching conclusions in other fields not related to geometry." To focus attention on this fact alone is a contribution of genuine value. Dr. Hellmich's recommendations seem sound

and sensible. They have far reaching implications with respect to the secondary school program and merit the most careful consideration of teachers in both fields.

H. P. Fawcett

Men of Mathematics. By E. T. Bell. Simon and Schuster, New York, 1937. xxi+592 pp. $5.00.

This very interesting new book is not a history of mathematics in the strict sense although the story deals especially with those mathematicians who created the Golden Age since Newton. The main interest for the reader throughout the book lies in the personalities of the men described.

Men of Mathematics should be accessible not only to those with a special interest in mathematics but to the general reader who wishes to understand how the race has progressed through the years. It is strange that historians, for example, have given so much attention to the war lords and their achievements in view of the damage they have done to civilization and have said so little about the great men of mathematics and science like Newton and Pasteur to whom we are greatly indebted. This book will make available a rich source of material for all those who wish to change the emphasis from men of war to men of peaceful pursuits.

This book can be readily understood by those who have a substantial background of secondary mathematics and will constitute a source of great enjoyment to many general readers because of the way in which Professor Bell presents his material. He is very observing, witty, and ironical and writes his story in a very fascinating style. His chapter on Galois, "Genius and Stupidity" is a classic.

Those who wish to understand more clearly the gradual growth and development of mathematics and the place that the subject has held in the lives of past generations will want to read this book. Again, it is unfortunate that the price of the book makes it almost inaccessible to the class room teacher of mathematics who obviously might, if he could read the book, pass on to his pupils the best of its content.

W. D. R.

Mathematics for Modern Life. By Jos. P. McCormack. D. Appleton-Century Company, New York, 1937. xv+448 p. $1.32.

This book is one of a large number of new texts designed primarily to meet the growing demand for something different from traditional algebra for ninth grade pupils, particularly those of lower mentality who seem destined to have only one year in which to get whatever mathematics a well educated citizen ought to know. The author claims to have applied "the newer principles of integrated teaching," whatever they are, and in doing this he deals with arithmetic with which the pupil is supposed to be familiar and also with the other topics of high school mathematics of the more advanced type such as the formula, graph, geometry, numerical trigonometry, logarithms, and the slide rule.

The book is well written, is illustrated at many points, and the material is very well ordered. Although fairly free from formalized algebra, the book would be improved by the omission of certain obsolete factoring types and some unnecessary polynomial operations. Teachers who are interested in these new one-year courses for the ninth grade will want to examine this new book.

W. D. R.

Plane Trigonometry by Edward S. Allen. McGraw-Hill Book Company, New York, 1936. xii+152 pp. Tables xxiii+156 pp. Price $2.25. The same book without tables, price $1.50.

New trigonometry textbooks in this country are not rare phenomena. Good trigonometries, however, are. But here we should be careful to clarify the meaning of the word "good." A *good mathematics textbook* is one that contains material correct mathematically, one that uses language free from ambiguities, one that is teachable. If a textbook is to be commended we cannot escape from these requirements.

The book reviewed here has many good points. It is written in a language that is simple, but perhaps this is partly the undoing of the author. In many cases this leads to inaccuracies of statement. For example, on page fifty we read "Trigonometric functions as we have seen are fractions." This is unfortunate, the word "ratio" would be closer to the truth. On page 126 we read "i is a number whose square is -1." This would stand revision. i is the symbol for that number whose square is -1. On the same page we also read, "An *irrational* number is one which, though not rational, can be approximated as closely as may be demanded by a rational number, e.g., π, $\sqrt{3}$." Editorial perusal should include "as closely as may be demanded" in commas. But even this would not make this definition mathematically acceptable.

The treatment of complex numbers in "polar" representation is debatable. The author entirely disregards the form (ρ, θ). Klein's definition of multiplication, that is, "we do to the number the same that was done to unity in order to obtain that number" should have enabled the introduction of graphical representation of the

process of multiplication, that is, the use of rotation and stretching.

The closing paragraph on page 138 is so ambiguous that it might as well be omitted. Surely the fundamental theorem of algebra is not limited to polynomial equations in x with complex coefficients only. Unfortunately this paragraph leads to this impression.

It is very questionable whether there is any justification for the inclusion of tables of Gamma Function and Probability Integral in the Tables. No student of trigonometry while studying it (even in an engineering school) will ever have an opportunity to use them. The same might be said about the tables of Natural Logarithms and of Hyperbolic Functions. The author makes no use of these tables in the text.

The computational aspects of the problems and of the illustrative problems in the book suffers much because the author disregards the importance of the fact that all data used in practical trigonometry is generally approximate. The author's attempt to elucidate this in a separate section is commendable but the effect of his discussion is totally lost in the maze of traditional treatment of problem material.

The solution of triangles phase is treated in this book well and it is presented in a manner that the student would find no difficulty in mastering.

A. Bakst

Scientific Inference. By Harold Jeffreys. The Macmillan Company, 1937. vii+272 pp. Price $3.25.

This book was first published in 1931 and is now re-issued with addenda and corrigenda. It deals with the philosophical problems, underlying scientific experimentation, such as the following: logic and scientific inference, probability, sampling, quantitative laws, errors, physical magnitudes, mensuration, Newtonian dynamics, light and relativity, and probability in logic and in pure mathematics.

The author writes in a style free from obscurity and replete with apt illustrations from science and everyday life. The latter characteristic can probably be ascribed to his professional preoccupation with the experimental side of geophysics.

It is to be regretted, however, that in the re-issue of his book the author did not find it convenient to review critically the contributions to the problems of meaning, scientific method, and probability that have been made in recent years by the logical—positivists in Europe and the United States.

N. Lazar